edexcel

GCE

Edexcel GCE

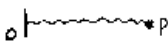Mathematics

Mechanics 3 M3 (6679

June 2008

advancing learning, changing lives

Mark Scheme (Final)
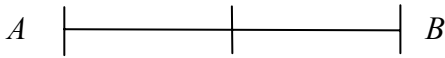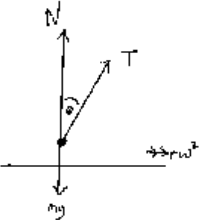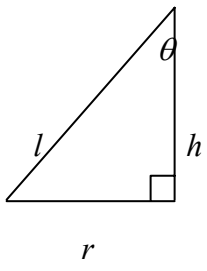
Edexcel GCE

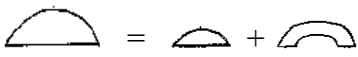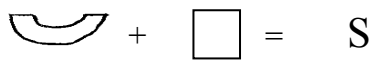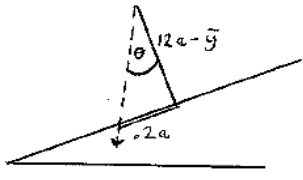Mathematics

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
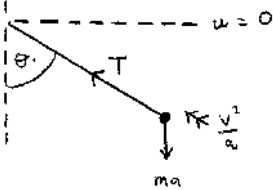
Question Number	Scheme	Marks
Q1(a)	EPE stored $= \dfrac{1}{2}\dfrac{\lambda}{L}\left(\dfrac{1}{2}L\right)^2 \quad \left(=\dfrac{\lambda L}{8}\right)$	B1
	KE gained $= \dfrac{1}{2}m\,2gL \quad (=mgL)$	B1
	EPE $=$ KE $\Rightarrow \dfrac{\lambda L}{8} = \text{mg}\,L \qquad$ i.e. $\lambda = 8mg$*	M1A1cso
		(4)
(b)	EPE $=$ GPE $+$ KE	M1
	$\dfrac{1}{2}\dfrac{8mg}{L}\left(\dfrac{1}{2}L\right)^2 = \dfrac{8mgL}{8} = mg\dfrac{L}{2} + \dfrac{1}{2}mu^2$	A1A1
	$\dfrac{mgL}{2} = \dfrac{m}{2}u^2 \quad \therefore u = \sqrt{gL}$	M1A1 **(5)**
		9 Marks

Question Number	Scheme	Marks
Q2 (a)		

$$O$$

$$A \;\vdash\!\!\!\!\!\!\!\!\longrightarrow\!\!\!\!\!\!\!\!\dashv\; B$$

$$T = 3 = \frac{2\pi}{\omega} \quad \therefore \; \omega = \frac{2\pi}{3}$$

M1A1

$$u^2 = \omega^2 \left(a^2 - x^2\right) \; ; \; a = 0.12 \quad , \quad u^2 = a^2\,\omega^2 \,, u = 0.12 \times \omega$$
$$= 0.251 \text{ ms}^{-1} \;\; (0.25 \text{ m s}^{-1})$$

M1

A1   **(4)**

(b)

Time from $O \rightarrow A \rightarrow O \; = 1.5\text{s}$ $\qquad\qquad \therefore \; t = 0.5$ B1

$$x = a \sin \omega t \qquad \Rightarrow OP = 0.12 \sin\left(\frac{\pi}{3}\right)$$ M1A1

Distance from B is $\quad 0.12 - OP = 0.12 - 0.104... = 0.016\text{m}$ M1A1 **(5)**

(c)

$$v^2 = \omega^2 \left(a^2 - x^2\right)$$ M1

$$v = \frac{2\pi}{3} \; \sqrt{0.12^2 - 0.104....^2} = \frac{2\pi}{3} \times 0.0598 = 0.13 \text{ ms}^{-1}$$ A1   **(2)**

11 Marks

Question Number	Scheme	Marks
Q3 (a)		

$$\uparrow \qquad T\cos\theta + N = Mg \qquad \textbf{(1)}$$

$$\rightarrow \qquad T\sin\theta = mr\omega^2 \qquad \textbf{(2)}$$

M1A1

M1A1

$$\sin\theta = \frac{r}{l} \quad \text{from (2)} \quad T = ml\omega^2$$

sub into **(1)** $\quad ml\cos\theta\,\omega^2 + N = mg$

M1

$$N = mg - mh\omega^2$$

A1

Since in contact with table $N \geq \textbf{0} \quad \therefore \omega^2 \leq \dfrac{g}{h}$ *****

M1A1 cso

(8)

(b)

$$r : h : l = 3 : 4 : 5 \quad \therefore \text{ extension} = \frac{h}{4}$$

B1

$$T = \frac{2mg}{h} \times \frac{h}{4} \quad = \frac{mg}{2}$$

M1A1

$$T = ml\omega^2 = \frac{5mh}{4}\omega^2 \quad \omega = \sqrt{\frac{2g}{5h}}$$

M1A1

(5)

13 marks

Question Number	Scheme	Marks
Q4 (a)	$\quad = \quad$ $+$	

Mass $a^3 \dfrac{2}{3}\pi \times$: $\quad$ 216 $\qquad$ 8 $\qquad$ 208 $\hfill$ M1A1

$\qquad\qquad\qquad\qquad$ 27 $\qquad$ 1 $\qquad$ 26

C of M from O: $\quad \dfrac{3}{8}\times 6a \qquad \dfrac{3}{8}\times 2a \qquad \overline{x}$ $\hfill$ Use of $\dfrac{3}{8}r$ $\hfill$ M1

Moment : $\quad 216 \times \dfrac{6a \times 3}{8} = 8 \times \dfrac{2a \times 3}{8} + 208\overline{x}$ $\hfill$ M1

$\qquad\qquad\qquad \overline{x} = \dfrac{480a}{208} = \dfrac{30a}{13} *$ $\hfill$ A1 cso $\hfill$ **(5)**

(b) $\qquad$ $+$ $\square$ $=$ S

Mass $\quad \pi a^3 \times$: $\quad \dfrac{416}{3} \quad + \quad 24 \quad = \quad \dfrac{488}{3}$ $\hfill$ B1

C of M: $\qquad \dfrac{30}{13}a \quad + \quad 9a \quad = \quad \overline{y}$ $\hfill$ B1

Moments : $\qquad 320a \quad + \quad 216a \quad = \quad \dfrac{488}{3}\overline{y}$ $\hfill$ M1

$\qquad\qquad\qquad \overline{y} = \dfrac{201}{61}a \, *$ $\hfill$ A1 cso $\hfill$ **(4)**

(c) $\qquad \tan\theta = \dfrac{2a}{12a - \frac{201}{61}a} \qquad \tan\theta = \dfrac{2a}{....}$ $\hfill$ $12a - \dfrac{201}{61}a$ $\hfill$ M1

$\hfill$ M1

$\qquad\qquad\qquad\qquad \theta = 12.93.....$ $\hfill$ A1

so critical angle = 12.93… $\qquad\qquad \therefore$ if $\theta = 12°$ it will <u>NOT</u> topple. $\hfill$ A1√ $\hfill$ **(4)** $\hfill$ **13 marks**

Question Number	Scheme	Marks
Q5(a)	Energy $\dfrac{1}{2}mv^2 = mga\cos\theta$ $v^2 = 2ga\cos\theta$	M1A1
	$F = ma \quad \nwarrow \quad T - mg\cos\theta = \dfrac{mv^2}{a}$	M1A1
	Sub for $\dfrac{v^2}{a}$: $\quad T = mg\cos\theta + 2mg\cos\theta$: $\quad \theta = 60 \quad \therefore T = \dfrac{3}{2}mg$	M1A1
		(6)
(b)	Speed of P before impact $= \sqrt{2ga}$	B1
	$\rightarrow \sqrt{2ga} \qquad \rightarrow 0 \qquad\qquad \rightarrow u$ PCLM : $\quad \bullet \qquad\quad \bullet \qquad \rightarrow \quad \bullet \qquad\qquad \therefore u = \dfrac{\sqrt{2ga}}{4} = \sqrt{\dfrac{ga}{8}} \; *$ $\qquad\quad$ m $\qquad\quad$ 3m $\qquad\quad$ 4m	M1A1cso
		(3)
(c) (i)	At A $v = 0$ so conservation of energy gives: $\qquad\qquad \dfrac{1}{2}\,4mu^2 = 4m\,ga\,(1 - \cos\theta)$	M1A1
	$\qquad\qquad \dfrac{ga}{16} = ga\,(1 - \cos\theta)$	M1
	$\qquad\qquad \cos\theta = \dfrac{15}{16} \;\; , \; \theta = 20^\circ$	A1
(ii)	$\nearrow \quad$ At $A \quad T = 4mg\cos\theta = \dfrac{15mg}{4} \quad$ (accept 3.75mg)	M1A1 **(6)**
		15 Marks

Question Number	Scheme	Marks
Q6 (a)	$F = ma \ (\rightarrow) \quad \dfrac{3}{(x+1)^3} = 0.5a = 0.5 \ v\dfrac{dv}{dx}$	M1A1
	$\displaystyle\int \dfrac{3}{(x+1)^3} \, dx = 0.5 \int v \, dv \qquad\qquad$ Separate and $\int$	M1
	$-\dfrac{3}{2(x+1)^2} = \dfrac{1}{4} \ v^2 \ (+ \ c)$	A1
	$x = 0, \ v = 0 \Rightarrow c' = -\dfrac{3}{2} \qquad \therefore \qquad v^2 = 6\left(1 - \dfrac{1}{(x+1)^2}\right)$ ✱	M1A1 cso **(6)**
(b)	$\forall x \quad v^2 < 6 \quad \therefore \quad v < \sqrt{6} \quad (\because (x+1)^2 \text{ always} > 0)$	B1 **(1)**
(c)	$v = \dfrac{dx}{dt} = \dfrac{\sqrt{6}\sqrt{(x+1)^2 - 1}}{x+1}$	M1
	$\displaystyle\int \dfrac{x+1}{\sqrt{(x+1)^2 - 1}} \, dx = \sqrt{6}\int dt$	M1
	$\sqrt{(x+1)^2 - 1} = \sqrt{6}\,t + c'$	M1 A1
	$t = 0, \ x = 0 \Rightarrow c' = 0$	M1
	$t = 2 \Rightarrow (x+1)^2 - 1 = (2\sqrt{6})^2$	M1
	$(x+1)^2 = 25 \quad \Rightarrow x = 4 \quad (c' \text{ need not have been found})$	A1 cao
		(7)
		14 Marks